

## **CLAIMS**

What is claimed is:

1. A method comprising:

providing a first texture map for a first portion of a three-dimensional surface, the first texture map being associated with a first mapping technique; and

providing a second texture map for a second portion of the three-dimensional surface, the second texture map being associated with a second mapping technique that is different from the first mapping technique.

2. The method as recited in Claim 1, wherein the first texture map includes cylindrical projection information for the first portion, and the second texture map includes azimuthal projection information for the second portion.

3. The method as recited in Claim 2, the method further comprising:

providing a third texture map for a third portion of the three-dimensional surface, the third texture map being associated with the second mapping technique, and wherein the third texture map includes azimuthal projection information for the third portion.

4. The method as recited in Claim 3, wherein the cylindrical projection information includes plane-chart projection information.

5. The method as recited in Claim 3, wherein the azimuthal projection information includes equidistant projection information.

6. The method as recited in Claim 2, wherein the first and second texture maps are stretch-invariant and have a sampling requirement definable as:

$$M_{\text{capped}}(\theta) \equiv M_{\text{equi}}(\theta) + M_{\text{plane}}(\pi/2 - \theta) = 4\theta^2 + 2\pi(\pi/2 - \theta)$$

where  $\theta$  is a transition angle from a defined point on the surface to where the second texture map is adjacent to the first texture map.

7. The method as recited in Claim 6, wherein  $\theta$  is equal to about  $45^\circ$ .

8. The method as recited in Claim 4, wherein providing the first texture map further includes hexagonally re-parameterizing the cylindrical projection information using a linear transform.

9. The method as recited in Claim 8, wherein the linear transform is definable as:

$$\hat{\mathbf{s}}(u, v) \equiv \mathbf{s}(\mathbf{V}(u, v)')$$

where

$$\mathbf{V} \equiv \begin{bmatrix} k & k/2 \\ 0 & 1 \end{bmatrix}$$

and  $k = 2\sqrt{3}/3$ .

10. The method as recited in Claim 1, wherein the first texture map includes Mercator projection information for the first portion, and the second texture map includes stereographic projection information for the second portion.

11. The method as recited in Claim 10, the method further comprising:  
providing a third texture map for a third portion of the three-dimensional surface, the third texture map being associated with the second mapping technique, and wherein the third texture map includes stereographic projection information for the third portion.

12. The method as recited in Claim 10, wherein the first and second texture maps are conformal and have a sampling requirement definable as:

$$M_{\text{capped}}(\theta) \equiv M_{\text{stereo}}(\theta) + M_{\text{Mercator}}(\pi/2 - \theta) = 16 \tan^2(\theta/2) + \pi \ln((1 + \cos \theta)/(1 - \cos \theta))$$

where  $\theta$  is a transition angle from a defined point on the surface to where the second texture map is adjacent to the first texture map.

13. The method as recited in Claim 12, wherein  $\theta$  is equal to about 47.8°

14. The method as recited in Claim 3, wherein the cylindrical projection information includes information selected from at least one type of projection information selected from a group comprising plane-chart projection information, equal area information, and Mercator information.

15. The method as recited in Claim 3, wherein the azimuthal projection information includes information selected from at least one type of projection information selected from a group comprising equidistant projection information, stereographic projection information, gnomonic projection information, and equal area projection information.

16. The method as recited in Claim 3, wherein the first portion is significantly adjacent to both the first and second portions, such that the first portion separates the second and third portions.

17. The method as recited in Claim 1, wherein the three-dimensional surface is curvilinear.

18. The method as recited in Claim 1, wherein the three-dimensional surface includes a spherical surface.

19. The method as recited in Claim 1, wherein providing the first texture map further includes generating the first texture map using the first mapping technique, and providing the second texture map further includes generating the second texture map using the second mapping technique.

20. The method as recited in Claim 1, wherein at least one of the first and second texture maps includes information based on a rectangular sampling matrix.

21. The method as recited in Claim 1, wherein at least one of the first and second texture maps includes information based on a hexagonal sampling matrix.

22. A computer-readable medium providing computer instructions suitable for performing steps comprising:

providing a first texture map for a first portion of a three-dimensional surface, the first texture map being associated with a first mapping technique; and

providing a second texture map for a second portion of the three-dimensional surface, the second texture map being associated with a second mapping technique that is different from the first mapping technique.

23. The computer-readable medium as recited in Claim 22, wherein the first texture map includes cylindrical projection information for the first portion, and the second texture map includes azimuthal projection information for the second portion.

24. The computer-readable medium as recited in Claim 23, further comprising computer instructions suitable for performing the step of:

providing a third texture map for a third portion of the three-dimensional surface, the third texture map being associated with the second mapping technique, and wherein the third texture map includes azimuthal projection information for the third portion.

25. The computer-readable medium as recited in Claim 24, wherein the cylindrical projection information includes plane-chart projection information.

26. The computer-readable medium as recited in Claim 24, wherein the azimuthal projection information includes equidistant projection information.

27. The computer-readable medium as recited in Claim 23, wherein the first and second texture maps are stretch-invariant and have a sampling requirement definable as:

$$M_{\text{capped}}(\theta) \equiv M_{\text{equi}}(\theta) + M_{\text{plane}}(\pi/2 - \theta) = 4\theta^2 + 2\pi(\pi/2 - \theta)$$

where  $\theta$  is a transition angle from a defined point on the surface to where the second texture map is adjacent to the first texture map.

28. The computer-readable medium as recited in Claim 27, wherein  $\theta$  is equal to about 45°.

29. The computer-readable medium as recited in Claim 25, wherein providing the first texture map further includes means for hexagonally re-parameterizing the cylindrical projection information using a linear transform.

30. The computer-readable medium as recited in Claim 29, wherein the linear transform is definable as:

$$\hat{\mathbf{s}}(u,v) \equiv \mathbf{S}(\mathbf{V}(u,v)')$$

where

$$\mathbf{V} \equiv \begin{bmatrix} k & k/2 \\ 0 & 1 \end{bmatrix}$$

and  $k \equiv 2\sqrt{3}/3$ .

31. The computer-readable medium as recited in Claim 22, wherein the first texture map includes Mercator projection information for the first portion, and

the second texture map includes stereographic projection information for the second portion.

32. The computer-readable medium as recited in Claim 31, further comprising computer instructions suitable for performing the step of:

providing a third texture map for a third portion of the three-dimensional surface, the third texture map being associated with the second mapping technique, and wherein the third texture map includes stereographic projection information for the third portion.

33. The computer-readable medium as recited in Claim 31, wherein the first and second texture maps are conformal and have a sampling requirement definable as:

$$M_{\text{capped}}(\theta) = M_{\text{stereo}}(\theta) + M_{\text{Mercator}}(\pi/2 - \theta) = 16 \tan^2(\theta/2) + \pi \ln((1 + \cos \theta)/(1 - \cos \theta))$$

where  $\theta$  is a transition angle from a defined point on the surface to where the second texture map is adjacent to the first texture map.

34. The computer-readable medium as recited in Claim 33, wherein  $\theta$  is equal to about  $47.8^\circ$

35. The computer-readable medium as recited in Claim 24, wherein the cylindrical projection information includes information selected from at least one type of projection information selected from a group comprising plane-chart projection information, equal area information, and Mercator information.

36. The computer-readable medium as recited in Claim 24, wherein the azimuthal projection information includes information selected from at least one type of projection information selected from a group comprising equidistant projection information, stereographic projection information, gnomonic projection information, and equal area projection information.

37. The computer-readable medium as recited in Claim 24, wherein the first portion is significantly adjacent to both the first and second portions, such that the first portion separates the second and third portions.

38. The computer-readable medium as recited in Claim 22, wherein the three-dimensional surface is curvilinear.

39. The computer-readable medium as recited in Claim 22, wherein the three-dimensional surface includes a spherical surface.

40. The computer-readable medium as recited in Claim 22, wherein providing the first texture map further includes generating the first texture map using the first mapping technique, and providing the second texture map further includes generating the second texture map using the second mapping technique.

41. The computer-readable medium as recited in Claim 22, wherein at least one of the first and second texture maps includes information based on a rectangular sampling matrix.

42. The computer-readable medium as recited in Claim 22, wherein at least one of the first and second texture maps includes information based on a hexagonal sampling matrix.

43. An apparatus comprising:

logic configured to provide a first texture map for a first portion of a three-dimensional surface, the first texture map being associated with a first mapping technique and a second texture map for a second portion of the three-dimensional surface, the second texture map being associated with a second mapping technique that is different from the first mapping technique, and wherein the logic is further configured to output graphically displayable information based on at least a portion of the first and second texture maps.

44. The apparatus as recited in Claim 43, wherein the first texture map includes cylindrical projection information for the first portion, and the second texture map includes azimuthal projection information for the second portion.

45. The apparatus as recited in Claim 44, wherein the logic is further configured to provide a third texture map for a third portion of the three-dimensional surface, the third texture map being associated with the second mapping technique, and wherein the third texture map includes azimuthal projection information for the third portion.

46. The apparatus as recited in Claim 45, wherein the cylindrical projection information includes plane-chart projection information.

47. The apparatus as recited in Claim 45, wherein the azimuthal projection information includes equidistant projection information.

48. The apparatus as recited in Claim 44, wherein the first and second texture maps are stretch-invariant and have a sampling requirement definable as:

$$M_{\text{capped}}(\theta) \equiv M_{\text{equi}}(\theta) + M_{\text{plane}}(\pi/2 - \theta) = 4\theta^2 + 2\pi(\pi/2 - \theta)$$

where  $\theta$  is a transition angle from a defined point on the surface to where the second texture map is adjacent to the first texture map.

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49. The apparatus as recited in Claim 48, wherein  $\theta$  is equal to about  $45^\circ$ .

50. The apparatus as recited in Claim 46, wherein the cylindrical projection information in the first texture map has been hexagonally re-parameterized the using a linear transform.

51. The apparatus as recited in Claim 50, wherein the linear transform is definable as:

$$\hat{\mathbf{s}}(u,v) \equiv \mathbf{s}(\mathbf{v}(u,v)')$$

where

$$\mathbf{v} \equiv \begin{bmatrix} k & k/2 \\ 0 & 1 \end{bmatrix}$$

and  $k \equiv 2\sqrt{3}/3$ .

52. The apparatus as recited in Claim 43, wherein the first texture map includes Mercator projection information for the first portion, and the second texture map includes stereographic projection information for the second portion.

53. The apparatus as recited in Claim 52, wherein the logic is further configured to provide a third texture map for a third portion of the three-dimensional surface, the third texture map being associated with the second mapping technique, and wherein the third texture map includes stereographic projection information for the third portion.

54. The apparatus as recited in Claim 52, wherein the first and second texture maps are conformal and have a sampling requirement definable as:

$$M_{\text{capped}}(\theta) = M_{\text{stereo}}(\theta) + M_{\text{Mercator}}(\pi/2 - \theta) = 16 \tan^2(\theta/2) + \pi \ln((1+\cos\theta)/(1-\cos\theta))$$

where  $\theta$  is a transition angle from a defined point on the surface to where the second texture map is adjacent to the first texture map.

55. The apparatus as recited in Claim 54, wherein  $\theta$  is equal to about  $47.8^\circ$ .

56. The apparatus as recited in Claim 45, wherein the cylindrical projection information includes information selected from at least one type of projection information selected from a group comprising plane-chart projection information, equal area information, and Mercator information.

57. The apparatus as recited in Claim 45, wherein the azimuthal projection information includes information selected from at least one type of projection information selected from a group comprising equidistant projection information, stereographic projection information, gnomonic projection information, and equal area projection information.

58. The apparatus as recited in Claim 45, wherein the first portion is significantly adjacent to both the first and second portions, such that the first portion separates the second and third portions.

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59. The apparatus as recited in Claim 43, wherein the three-dimensional surface is curvilinear.

60. The apparatus as recited in Claim 43, wherein the three-dimensional surface includes a spherical surface.

61. The apparatus as recited in Claim 43, wherein the logic is further configured to analyze the texture map per at least one criterion to determine an appropriate texture resolution when providing the first texture map.

62. The apparatus as recited in Claim 43, wherein the logic is further configured to analyze the texture map per at least one metric criterion to determine a requisite number of texture maps in addition to the first texture map when providing the first texture map.

63. The apparatus as recited in Claim 43, wherein at least one of the first and second texture maps includes information based on a rectangular sampling matrix.

64. The apparatus as recited in Claim 43, wherein at least one of the first and second texture maps includes information based on a hexagonal sampling matrix.

65. A polar-capped map set for use in computer generated graphics, the polar-capped map set comprising:

- a cylindrical projection map; and
- at least one azimuthal projection map.

66. The polar-capped map as recited in Claim 65, wherein the polar-capped map is a stretch-invariant map.

67. The polar-capped map as recited in Claim 65, wherein the polar-capped map is a conformal map.

68. A method for generating a low-distortion area-preserving map for use in stochastic ray tracing computer generated graphics, the method comprising:  
projecting sampling patterns onto a three-dimensional surface; and  
projecting the resulting three-dimensional surface samples into two-dimensional histogram bins.

69. The method as recited in Claim 68, wherein projecting the sampling patterns includes a projection,  $(u,v) = \mathbf{S}^{-1}(x,y,z)$ , that is defined by the composition of at least two area-preserving bijections, wherein a first area-preserving bijection is a mapping from a hemisphere to a disk  $(u,v) = (x,y)/\sqrt{1+z}$ , a second area-preserving bijection is from a disk to a half disk  $(r',\theta') = (r,\theta/2)$ .

70. The method as recited in Claim 1, wherein providing the first texture map further includes analyzing the texture map per at least one criterion to determine an appropriate texture resolution.

71. The method as recited in Claim 1, wherein providing the first texture map further includes analyzing the texture map per at least one metric criterion to determine a requisite number of texture maps in addition to the first texture map.

72. The computer-readable medium as recited in Claim 22, wherein providing the first texture map further includes analyzing the texture map per at least one criterion to determine an appropriate texture resolution.

73. The computer-readable medium as recited in Claim 22, wherein providing the first texture map further includes analyzing the texture map per at least one metric criterion to determine a requisite number of texture maps in addition to the first texture map.